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RELATIONS BETWEEN FORCES ACTING ON BODIES OF DIFFERENT SHAPES MOVING IN A GAS *

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A method of constructing three-dimensional bodies with identical drag coefficients in a streamlined flow under the conditions of the hypothesis of local interaction, is investigated. The equality of the characteristics is maintained irrespective of the choice of the functions describing the interaction between the flow and the body surface, i.e. on changing the mode of the flow.

1. The authors of /1,2/ gave a method of computing aerodynamic characteristics of bodies in a flow under the conditions of the hypothesis of local interaction, i.e. under the assumption that the local force coefficient c_f acting from the direction of the flow on the body at the given point of its surface, depends only on the local angle between the direction of the incoming flow $\mathbf{v}$ and the inner normal unit vector $\mathbf{n}$

$$\mathbf{c}_f = \Omega_p(\mathbf{v}\cdot\mathbf{n})\cdot\mathbf{n} + \Omega_\tau(\mathbf{v}\cdot\mathbf{n})\cdot\boldsymbol{\tau}, \quad \boldsymbol{\tau} = [\mathbf{v} - \mathbf{n}(\mathbf{v}\cdot\mathbf{n})] / \sqrt{1-(\mathbf{v}\cdot\mathbf{n})^2} \tag{1.1}$$

where the functions Ω_p and Ω_τ define the actual model of the flow (the parameters characterizing the flow around the body can also be used as the arguments).

The method developed in /1,2/ makes it possible to construct, for the given functions Ω_p and Ω_τ, classes of the corresponding bodies the characteristic features of which are connected by linear relationships. The authors discussed plane profiles, solids of revolution and some types of the three-dimensional bodies. The distinctive features of the approach adopted in the present paper, which extend appreciably its range of application, are as follows:change to investigating the three-dimensional bodies, and the refusal to specify a concrete form of the functions Ω_p and Ω_τ. We show that a class of the corresponding solids can be constructed for a three-dimensional body, characterized by identical drag coefficient in the flow, and this equality is preserved when the form of the functions Ω_p and Ω_τ is changed, i.e. when the mode of the flow is changed within the framework of the hypothesis of local interaction. As a result, a possibility arises of computing the characteristics of the corresponding bodies using the results of experiments carried out for the parent body, without specifying the models of flow. The parent body and the corresponding bodies have, in spite of appreciable variation in form, identical drag coefficients (over a wide range of the flow modes), and this widens the range of choice of a reasonable form of aircraft.

2. We write the expression for the body drag in the form

$$F = S_0 q c_F = q \iint\limits_{(S)} \mathbf{c}_f\cdot\mathbf{v}\, dS = q\iint\limits_{(S)} \Omega(\mathbf{v}\cdot\mathbf{n})\, dS, \quad \Omega(t) = t\Omega_p(t) + \sqrt{1-t^2}\,\Omega_\tau(t) \tag{2.1}$$

where S_0 denotes the characteristic area, c_F is the drag coefficient of the body and q denotes the dynamic head.

Let us consider two bodies, $T^{(0)}$ and $T^{(1)}$ (here and henceforth the superscripts in round brackets denote the number of the body). Let the form of the body $T^{(i)}$ be defined in Cartesian coordinates by the vector $\mathbf{r}^{(i)}(\alpha, \beta)$, with $(\alpha, \beta) \in (\sigma_{\alpha\beta})$. Then the expression for the resistance force can be written in the form (the $\pm$ sign determines the choice of the inner normal to the surface)

$$F^{(i)} = S_0^{(i)} q c_F^{(i)} = q \iint\limits_{(\sigma_{\alpha\beta})} \Omega\left[\pm\frac{\mathbf{v}^{(i)}\mathbf{r}_\alpha^{(i)}\mathbf{r}_\beta^{(i)}}{|\mathbf{r}_\alpha^{(i)}\times\mathbf{r}_\beta^{(i)}|}\right] |\mathbf{r}_\alpha^{(i)}\times\mathbf{r}_\beta^{(i)}|\, d\alpha\, d\beta \tag{2.2}$$

Our aim is to determine the relation between the forms of the surfaces of $T^{(0)}$ and $T^{(1)}$ ensuring that the ratio of their drags

$$F^{(1)} / F^{(0)} = k \tag{2.3}$$

remains the same when the function Ω is varied. We shall call such bodies the corresponding bodies.

Let us first consider the simplest case when the form of the bodies is such that $\mathbf{v}^{(i)}\cdot\mathbf{n}^{(i)} = \mu = \text{const}$. Then from (2.2) it follows that condition (2.3) holds for these bodies and $k = S^{(1)} / S^{(0)}$ where $S^{(i)}$ is the area of the "wettable" surface (where $\Omega \neq 0$).

In the general case the condition (2.3) holds irrespective of the choice of Ω, provided that the following system of conditions is satisfied:

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$$|\mathbf{r}_\alpha^{(1)} \times \mathbf{r}_\beta^{(1)}| = k\,|\mathbf{r}_\alpha^{(0)} \times \mathbf{r}_\beta^{(0)}| \tag{2.4}$$
$$\frac{\mathbf{v}^{(1)}\mathbf{r}_\alpha^{(1)}\mathbf{r}_\beta^{(1)}}{|\mathbf{r}_\alpha^{(1)} \times \mathbf{r}_\beta^{(1)}|} = \pm \frac{\mathbf{v}^{(0)}\mathbf{r}_\alpha^{(0)}\mathbf{r}_\beta^{(0)}}{|\mathbf{r}_\alpha^{(0)} \times \mathbf{r}_\beta^{(0)}|}$$

From the first equation of (2.4) follows

$$S^{(1)} = kS^{(0)} \tag{2.5}$$

which connects the areas of the wettable surfaces of the corresponding bodies.

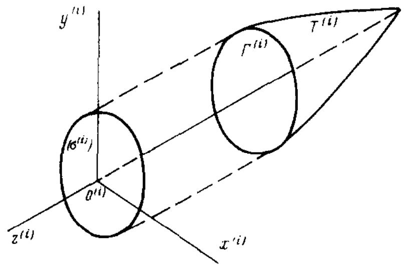

Fig.1

3. Let us now pass to the problem of determining the form of the corresponding body $T^{(0)}$ when the form of $T^{(1)}$ is known. We choose a coordinate system (Fig.1), attached to the body in such a manner that the directions of the $O^{(i)}z^{(i)}$ -axis and of $v^{(i)}$ coincide. The equation $z = z(x, y)$ determines the surface of $T^{(0)}$ (from now on the zero superscript will be omitted). The form of the surface of $T^{(1)}$ is described by the parametric equation $x^{(1)} = x^{(1)}(x, y)$, $y^{(1)} = y^{(1)}(x, y)$, $z^{(1)} = z^{(1)}[x^{(1)}(x, y), y^{(1)}(x, y)]$. Setting $\alpha = x$, $\beta = y$, we can write the conditions (2.4) in the form

$$x_x^{(1)}y_y^{(1)} - x_y^{(1)}y_x^{(1)} = k \tag{3.1}$$

$$z_x^2 + z_y^2 = [z_{x^{(1)}}^{(1)2} + z_{y^{(1)}}^{(1)2}]_{x^{(1)}=x^{(1)}(x,y),\ y^{(1)}=y^{(1)}(x,y)} \tag{3.2}$$
$$x_\delta^{(i)} = \partial x^{(i)}/\partial\delta,\ y_\delta^{(i)} = \partial y^{(i)}/\partial\delta,\ z_\delta^{(i)} = \partial z^{(i)}/\partial\delta,\ \delta = x, y, x^{(1)}, y^{(1)}$$

Let us denote the projection of the body surface described by a single-valued function $z^{(i)}(x^{(i)}, y^{(i)})$ on the plane $z^{(i)} = 0$, by $(\sigma^{(i)})$; $(\sigma^{(i)})$ also denotes the projection of the middle cross section of the body. Then the functions $x^{(1)}(x, y)$, $y^{(1)}(x, y)$ define the transformation of the region $(\sigma^{(0)})$ to $(\sigma^{(1)})$, and from (3.1) follows

$$\sigma^{(1)} = k\sigma^{(0)} \tag{3.3}$$

Comparing (2.2), (2.3) and (3.3), we can conclude that the drag coefficients of the corresponding bodies are the same, provided that the area of wettable surface or its projection on the plane perpendicular to the direction of flow is chosen as the characteristic area. The relations (2.3) and (2.5) can be used to calculate the effective Reynold's number for the corresponding body.

The process of finding the corresponding body is reduced to the following. Let the equation of the surface of $T^{(1)}$ bounded by the contour $\Gamma^{(1)}$ be specified; the projection of $\Gamma^{(1)}$ on the plane $z^{(1)} = 0$ (the closed curve $\gamma^{(1)}$) encloses the region $(\sigma^{(1)})$. The form of the middle cross section of the corresponding body $\Gamma^{(0)}$ is also specified. Then if the transformation $x^{(1)} = x^{(1)}(x, y)$, $y^{(1)} = y^{(1)}(x,y)$, taking $(\sigma^{(0)})$ into $(\sigma^{(1)})$ and $(\gamma^{(0)}$ into $\gamma^{(1)})$ has been obtained, then the surface of the corresponding body can be found by solving the equation (3.2); the surface should pass across the contour $\Gamma^{(0)}$ (Cauchy problem for the first order partial differential equations).

We note that various characteristics (e.g. other components of the aerodynamic force) can be used as F in (2.1), and it can be generalized to more general cases of representing F.

4. As an example, we shall consider a class of corresponding bodies generated by the transformation

$$x^{(1)} = Ax,\quad y^{(1)} = By,\quad A = a\sqrt{k},\quad B = \sqrt{k}/a$$

and assume that the contour $\Gamma^{(i)}$ surrounding the middle cross section of the body $T^{(i)}$ lies in the plane $z^{(i)} = 0$. Such a transformation takes, in particular, solids of revolution into the solids with an elliptical middle cross section.

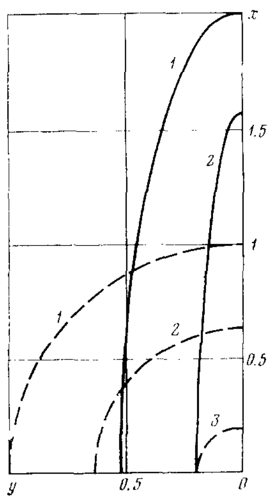

Fig.2

Let the parent body be formed by rotation of a parabolic arc. The equation describing its surface has the form

$$z^{(1)} = {}^1/_2 \left(\sqrt{x^{(1)2} + y^{(1)2}} + \theta\right)^2 - {}^1/_2 (1+\theta)^2$$

Then the equation of the surface of $T^{(0)}$ is obtained as a solution of the problem

$$z_x^2 + z_y^2 = A^2 x^2 + B^2 y^2$$
$$z = 0, \quad x = x_0(t), \quad y = y_0(t) \quad A^2 x_0^2 + B^2 y_0^2 = 1$$

The solution has the following form under the assumption that the body is thin in the direction of the Oy-axis

$$\bar{z}(\bar{x}, \bar{y}) = |\bar{y}| \left({}^1/_2 \sqrt{\bar{x}^2 + \bar{y}^2} + \theta\right) - \left({}^1/_2 - \theta\right) \sqrt{1 - \bar{x}^2} +$$
$$\bar{x}^2 \ln \frac{|\bar{y}| + \sqrt{\bar{x}^2 + \bar{y}^2}}{1 + \sqrt{1 - \bar{x}^2}}, \quad \bar{z} = Bz, \quad \bar{x} = Ax, \quad \bar{y} = By$$

The form of the bodies with the same drag is shown in Fig.2 ($k = 1$, $a = 0.5$, $\theta = 2.5$). All linear dimensions are referred to some characteristic length. Solid lines refer to the body $T^{(0)}$ and dashed line to $T^{(1)}$. The curves show the form of the cross sections of the surface of the body intersected by the plane $z^{(i)} = -d$ perpendicular to the direction of the incident flow ($x^{(i)} \geqslant 0$, $y^{(i)} \geqslant 0$) and correspond, in the order of increasing numbers, to the values of d equal to $0, 1$ and 2. It is apparent that the transformation changes substantially the form of the body, while maintaining the value of the drag.

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